In-medium electron-nucleon scattering

D. H. Lu, A. W. Thomas, K. Tsushima, A. G. Williams

Department of Physics and Mathematical Physics

and Special Research Centre for the Subatomic Structure of Matter,

University of Adelaide, Australia 5005

K. Saito

Physics Division, Tohoku College of Pharmacy,

Sendai 981, Japan

Abstract

In-medium nucleon electromagnetic form factors are calculated in the quark meson coupling model. The form factors are typically found to be suppressed as the density increases. For example, at normal nuclear density and $Q^2 \sim 0.3$ GeV^2, the nucleon electric form factors are reduced by approximately 8% while the magnetic form factors are reduced by only $1-2\%$. These variations are consistent with current experimental limits but should be tested by more precise experiments in the near future.

PACS numbers: 12.39, 21.65, 13.40.Gp

ADP97-17-T254

There is now considerable evidence that nucleons bound in an atomic nucleus experience very strong, effective scalar and vector fields [1–5]. It is a fundamental issue in nuclear physics to understand whether these strong fields alter the internal structure of the nucleon to a significant extent. There are already strong, experimental constraints on the extent of such changes [6–8]. Furthermore, new quasi-elastic (e, e') and $(e, e'N)$ experiments at facilities such as Mainz, MIT-Bates and especially TJNAF should improve these constraints in the near future. Our aim here is to examine the changes in the electromagnetic form factors of bound nucleons within one particular quark model of nuclear structure, the quark meson coupling (QMC) model [3–5,9]. In particular, we show that the model is consistent with existing constraints but that the predicted changes should be testable with only a modest increase in precision.

As in Quantum Hadrodynamics (QHD) [1], the QMC model describes the properties of nuclear systems using effective scalar (σ) and vector (ω) fields. However, unlike QHD where the nucleon is a point particle, the nucleon in this model has substructure. The σ and ω fields are coupled directly to the quarks within the nucleons, rather than to the nucleons themselves. As a result, the internal structure of the bound "nucleon" is modified by the medium with respect to the free case. The relative simplicity of the QMC model at the hadronic level, together with its phenomenological successes, makes it suitable for many applications in nuclear physics [10–12].

The structure of the nucleon in the QMC model is described by the MIT bag [13]. (However, there are also versions which use the color dielectric model [14] or a relativistic oscillator [10].) The small mass of the quark in these models implies that the lower component of the quark wave function responds rapidly to the change of its environment (σ field), with a consequent decrease in the scalar baryon density. As the scalar baryon density itself is the source of the σ field, this provides a mechanism for the saturation of nuclear matter where the quark structure plays a vital role.

In this article we use the self-consistent solutions of the QMC model to calculate the density-dependent electromagnetic form factors of the nucleon. For simplicity, possible off-

shell effects of the nucleon in medium [15] are ignored in the present treatment. The nucleon bag contains three independent quarks. With the static spherical cavity approximation to the MIT bag [13], the center-of-mass motion has to be corrected. We use the Peierls-Thouless projection method together with a Lorentz contraction for the nucleon internal wave function. This technique for implementing the center-of-mass and recoil corrections has been quite successful in the case of the electromagnetic form factors for free nucleons [16].

For an on-shell nucleon, the electric (G_E) and magnetic (G_M) form factors can be conveniently defined in the Breit frame by

$$\langle N_{s'}(\frac{\vec{q}}{2})|\, J^0(0)\, |N_s(-\frac{\vec{q}}{2})\rangle = \chi^\dagger_{s'}\, \chi_s\, G_E(Q^2), \tag{1}$$

$$\langle N_{s'}(\frac{\vec{q}}{2})|\, \vec{J}(0)\, |N_s(-\frac{\vec{q}}{2})\rangle = \chi^\dagger_{s'}\, \frac{i\boldsymbol{\sigma} \times \vec{q}}{2m_N}\, \chi_s\, G_M(Q^2), \tag{2}$$

where χ_s and $\chi^\dagger_{s'}$ are Pauli spinors for the initial and final nucleons respectively, $\vec{q}$ is the three momentum transfer, and $q^2 = -Q^2 = -\vec{q}^{\,2}$. The major advantage of the Breit frame is that G_E and G_M are explicitly decoupled and can be determined by the time and space components of the electromagnetic current (J^μ), respectively. Note that, in the above definitions [Eqs. (1) and (2)], both the initial and final states are physical states which incorporate meson clouds.

In the QMC model, the quarks typically move much faster than the nucleon in the medium (the Fermi motion). Thus it is reasonable to assume that they always have sufficient time to adjust their motion so that they stay in the lowest energy state [9]. In the instantaneous rest frame of a nucleon in infinite nuclear matter, the Lagrangian for the quark-meson coupling model is

$$\mathcal{L}_q = \overline{q}(i\gamma^\mu\partial_\mu - m_q)q\theta_V - B\theta_V + g^q_\sigma\overline{q}q\sigma - g^q_\omega\overline{q}\gamma_\mu q\omega^\mu - \frac{1}{2}m^2_\sigma\sigma^2 + \frac{1}{2}m^2_\omega\omega^2, \tag{3}$$

where m_q is the current quark mass, B is the bag constant, g^q_σ and g^q_ω denote the quark-meson coupling constants, and θ_V is a step function which is one inside the bag volume and vanishes outside. In mean field approximation for the meson field, the normalized solution

for the lowest state of the quark is given by [13]

$$q_m(t, \mathbf{r}) = \frac{N_0}{\sqrt{4\pi}} e^{-i\epsilon_q t/R} \begin{pmatrix} j_0(\omega_0 r/R) \\ i\beta_q \boldsymbol{\sigma} \cdot \hat{r} j_1(\omega_0 r/R) \end{pmatrix} \theta(R-r)\, \chi_m, \tag{4}$$

where

$$\epsilon_q = \Omega_q + g_\omega \overline{\omega} R, \qquad \beta_q = \sqrt{\frac{\Omega_q - m_q^* R}{\Omega_q + m_q^* R}}, \tag{5}$$

$$N_0^{-2} = 2R^3 j_0^2(\omega_0)[\Omega_q(\Omega_q - 1) + m_q^* R/2]/\omega_0^2, \tag{6}$$

with $\Omega_q \equiv \sqrt{\omega_0^2 + (m_q^* R)^2}$, $m_q^* \equiv m_q - g_\sigma^q \overline{\sigma}$, R the bag radius, and χ_m the quark Pauli spinor. The frequency (ω_0) of this lowest mode is determined by the boundary condition at the bag surface,

$$j_0(\omega_0) = \beta_q j_1(\omega_0). \tag{7}$$

The form of the quark wave function in Eq. (4) is almost identical to the free case. However the parameters in the solution, Eq. (4), have been modified by the medium. The mean values of the scalar field $\overline{\sigma}$ and vector field $\overline{\omega}$ are self-consistently determined by the coupled equations of motion for the system [3,5,9]. In particular, $\overline{\sigma}$ is given by the thermodynamic condition

$$\left.\frac{\partial E_{\mathrm{total}}}{\partial \overline{\sigma}}\right|_{R,\rho} = 0, \tag{8}$$

where ρ is the baryon density and the total energy per nucleon for symmetric nuclear matter is given by

$$E_{\mathrm{total}} = \frac{4}{(2\pi)^3 \rho} \int^{k_F} d^3k \sqrt{{m_N^*}^2 + k^2} + \frac{m_\sigma^2}{2\rho} \overline{\sigma}^2 + \frac{(3g_\omega^q)^2}{2m_\omega^2} \rho, \tag{9}$$

and $\overline{\omega}$ is determined through baryon number conservation to be

$$\overline{\omega} = \frac{3g_\omega^q}{m_\omega^2} \rho\, . \tag{10}$$

The effective mass of the nucleon in the medium can be expressed as

$$m_N^* = \frac{3\Omega_q}{R} - \frac{z_0}{R} + \frac{4}{3}\pi R^3 B, \tag{11}$$

where the second term on the right hand side of the above equation parameterizes the sum of the center of mass motion and gluon corrections. In the present work, the values of z_0 and B are assumed to be density-independent (the former for reasons explained in Ref. [9]). They are determined by requiring that the free nucleon mass be given by $m_N = 939$ MeV and by the equilibrium condition in free space for a given bag radius,

$$\left.\frac{\partial m_N^*}{\partial R}\right|_{\rho=0} = 0. \tag{12}$$

For finite nuclear density, the in-medium bag radius R can be obtained by solving Eq. (12) at finite ρ – it typically decreases by a few percent at nuclear matter density (ρ_0).

The electromagnetic current of the quark is simply

$$j_q^\mu(x) = \sum_f Q_f e \overline{q}_f(x)\gamma^\mu q_f(x), \tag{13}$$

where $q_f(x)$ is the quark field operator for the flavor f and Q_f is its charge in units of e. The momentum eigenstate of a baryon is constructed by the Peierls-Thouless projection method [16,17],

$$\Psi_{\rm PT}(\vec{x}_1, \vec{x}_2, \vec{x}_3; \vec{p}) = N_{\rm PT} e^{i\vec{p}\cdot\vec{x}_{\rm c.m.}} q(\vec{x}_1 - \vec{x}_{\rm c.m.}) q(\vec{x}_2 - \vec{x}_{\rm c.m.}) q(\vec{x}_3 - \vec{x}_{\rm c.m.}), \tag{14}$$

where $N_{\rm PT}$ is a normalization constant, $\vec{p}$ the total momentum of the baryon, and $\vec{x}_{\rm c.m.} = (\vec{x}_1 + \vec{x}_2 + \vec{x}_3)/3$ is the center of mass of the baryon (we assume equal mass quarks here).

Using Eqs. (13) and (14), the nucleon electromagnetic form factors for the proton's quark core can be easily calculated by

$$G_E(Q^2) = \int d^3r j_0(Qr)\rho_q(r)K(r)/D_{\rm PT}, \tag{15}$$

$$G_M(Q^2) = (2m_N/Q)\int d^3r j_1(Qr)\beta_q j_0(\omega_0 r/R) j_1(\omega_0 r/R)K(r)/D_{\rm PT}, \tag{16}$$

$$D_{\rm PT} = \int d^3r \rho_q(r)K(r), \tag{17}$$

where $D_{\rm PT}$ is the normalization factor, $\rho_q(r) \equiv j_0^2(\omega_0 r/R) + \beta_q^2 j_1^2(\omega_0 r/R)$, and $K(r) \equiv \int d^3x\, \rho_q(\vec{x})\rho_q(-\vec{x}-\vec{r})$ is the recoil function to account for the correlation of the two spectator quarks.

Apart from the center-of-mass correction, it is also vital to include Lorentz contraction of the bag for the form factors at moderate momentum transfer [16,18]. In the prefered Breit frame, the photon-quark interaction can be reasonably treated as instantaneous. The final form of the form factors can be obtained through a simple rescaling, i.e.,

$$G_E(Q^2) = (\frac{m_N^*}{E^*})^2 G_E^{\rm sph}(Q^2 {m_N^*}^2/E^{*2}), \tag{18}$$

$$G_M(Q^2) = (\frac{m_N^*}{E^*})^2 G_M^{\rm sph}(Q^2 {m_N^*}^2/E^{*2}), \tag{19}$$

where $E^* = \sqrt{{m_N^*}^2 + Q^2/4}$ and $G_{M,E}^{\rm sph}(Q^2)$ are the form factors calculated with the static spherical bag wave function [Eqs. (15) and (16)]. The scaling factor in the argument arises from the coordinate transformation of the struck quark and the factor in the front, $(m_N^*/E^*)^2$, comes from the reduction of the integral measure of two spectator quarks in the Breit frame [18].

As is well-known, a realistic picture of the nucleon should include the surrounding meson cloud. Following the cloudy bag model (CBM) [19,20], we limit our consideration on the meson cloud correction to the most important component, namely the pion cloud. As in free space, the pion field is a Goldstone boson and acts to restore the chiral symmetry. The Lagrangian related to the pion field and its interaction, within the pseudoscalar quark-pion coupling scheme, is

$$\mathcal{L}_{\pi q} = \frac{1}{2}(\partial_\mu \boldsymbol{\pi})^2 - \frac{1}{2} m_\pi^2 \boldsymbol{\pi}^2 - \frac{i}{2 f_\pi} \overline{q} \gamma_5 \boldsymbol{\tau} \cdot \boldsymbol{\pi} q \delta_S, \tag{20}$$

where δ_S is a surface delta function of the bag, m_π the pion mass and f_π the pion decay constant. The electromagnetic current of the pion is

$$j_\pi^\mu(x) = -ie[\pi^\dagger(x)\partial^\mu \pi(x) - \pi(x)\partial^\mu \pi^\dagger(x)], \tag{21}$$

where $\pi(x) = \frac{1}{\sqrt{2}}[\pi_1(x) + i\pi_2(x)]$ either destroys a negatively charged pion or creates a positively charged one. As long as the bag radius is above 0.7 fm, the pion field is relatively weak and can be treated perturbatively. A physical baryon state can be expressed as [20]

$$|A\rangle = \sqrt{Z_2^A}[1 + (m_A - H_0 - \Lambda H_I \Lambda)^{-1} H_I]\,|A_0\rangle, \tag{22}$$

where Λ is a projection operator which projects out all the components of $|A\rangle$ with at least one pion, H_I is the interaction Hamiltonian which describes the process of emission and absorption of pions. The matrix elements of H_I between the bare baryon states and their properties are [20]

$$v_{0j}^{AB}(\vec{k}) \equiv \langle A_0| H_I |\pi_j(\vec{k})B_0\rangle = \frac{i f_0^{AB}}{m_\pi} \frac{u(kR)}{[2\omega_k(2\pi)^3]^{1/2}} \sum_{m,n} C^{s_B m s_A}_{S_B 1 S_A}(\hat{s}^*_m \cdot \vec{k}) C^{t_B n t_A}_{T_B 1 T_A}(\hat{t}^*_n \cdot \vec{e}_j), \tag{23}$$

$$w_{0j}^{AB}(\vec{k}) \equiv \langle A_0 \pi_j(\vec{k})| H_I |B_0\rangle = \left[v_{0j}^{BA}(\vec{k})\right]^* = -v_{0j}^{AB}(\vec{k}) = v_{0j}^{AB}(-\vec{k}), \tag{24}$$

where the pion has momentum $\vec{k}$ and isospin projection j, f_0^{AB} is the reduced matrix element for the $\pi B_0 \to A_0$ transition vertex, $u(kR) = 3j_1(kR)/kR$, $\omega_k = \sqrt{k^2 + m_\pi^2}$, and $\hat{s}_m$ and $\hat{t}_n$ are spherical unit vectors for spin and isospin, respectively. The πNN form factor $[u(kR)]$ is fully determined by the model itself and depends only on the bag radius. The bare baryon probability in the physical baryon state, Z_2^A, is given by

$$Z_2^A = \left[1 + \sum_B \left(\frac{f_0^{AB}}{m_\pi}\right)^2 \frac{1}{12\pi^2} \mathrm{P}\int_0^\infty \frac{dk\, k^4 u^2(kR)}{\omega_k (m_A - m_B - \omega_k)^2}\right]^{-1}, \tag{25}$$

where P denotes a principal value integral.

Up to one pion loop, the corrections to the electromagnetic form factors arising from the pion cloud are described by the following two processes shown in Fig. 1(b) and 1(c). The detailed expressions for their contributions can be found in Ref. [16] with the following substitutions $m_\pi \to m_\pi^*$, $m_B \to m_B^*$, and $f_{\pi AB} \to f_{\pi AB}^*$.

In principle, the existence of the π and Δ inside the nuclear medium will also lead to some modification of their properties. Since the pion is well approximated as a Goldstone boson, the explicit chiral symmetry breaking is small in free space, and it should be somewhat smaller in nuclear medium [21]. While the pion mass would be slightly smaller in the medium, because the pion field has little effect on the form factors (other than G_{En}), we use $m_\pi^* = m_\pi$. As the Δ is treated on the same footing as the nucleon in the CBM, its mass should vary in a similar manner as the nucleon. Thus we assume that the in-medium and free space $N - \Delta$ mass splitting are approximately equal, i.e., $m_\Delta^* - m_N^* \simeq m_\Delta - m_N$. The physical πAB coupling constant is obtained by $f^{AB} \simeq \left(\frac{f_0^{AB}}{f_0^{NN}}\right) f^{NN}$. There are uncertain

corrections on the bare coupling constant f_0^{NN}, such as the nonzero quark mass and the correction for spurious center of mass motion. Therefore, we use the renormalized coupling constant in our calculation, $f^{NN} \simeq 3.03$, which corresponds to the usual πNN coupling constant, $f^2_{\pi NN} \simeq 0.081$. In the medium, the πNN coupling constant might be expected to decrease slightly due to the enhancement of the lower component of the quark wave function, but we shall ignore this density dependence in a first treatment and use $f^*_{\pi NN} \simeq f_{\pi NN}$.

Fig. 2 summarises the nucleon electromagnetic form factors in free space with the bag radius R = 1 fm. The 'dipole' refers to the standard dipole fit, $F_d(Q^2) = 1/(1 + Q^2/0.71 \text{ GeV}^2)^2$. The corrections for the center-of-mass motion and Lorentz contraction lead to significantly better agreement with data than was obtained in the original, static CBM calculations. For a detailed comparison with data we refer to Ref. [16]. The purpose of Fig. 2 here is simply to show that our model produces realistic form factors in free space.

Our main results, namely the density dependence of the form factors in matter, relative to those in free space, are shown in Fig. 3. The charge form factors are much more sensitive to the nuclear medium density than the magnetic ones. The latter are nearly one order of magnitude less sensitive. Increasing density obviously suppresses the electromagnetic form factors for small Q^2. For a fixed Q^2 (less than 0.3 GeV^2), the form factors decrease almost linearly with respect to the nuclear density, ρ . At $Q^2 \sim 0.3$ GeV^2, the proton and neutron charge form factors are reduced by roughly 5% and 6% for $\rho = 0.5\rho_0$, and 8% for the normal nuclear density, ρ_0; similarly, the proton and neutron magnetic form factors are 1% and 0.6% smaller for $\rho = 0.5\rho_0$, and 1.5% and 0.9% for the normal nuclear density.

The best experimental constraints on the changes in these form factors come from the analysis of y-scaling data. For example, in Fe the nucleon root-mean-square radius cannot vary by more than 3% [6]. However, in the kinematic range covered by this analysis, the eN cross section is predominantly magnetic, so this limit applies essentially to G_M. (As the electric and magnetic form factors contribute typically in the ratio 1:3 the corresponding limit on G_E would be nearer 10%.) For the QMC model considered here, the calculated increase in the root-mean-square radius of the magnetic form factors is less than 0.8% at ρ_0.

For the electric form factors the best experimental limit seems to come from the Coulomb sum-rule, where a variation bigger than 4% would be excluded [8]. This is similar in size to the variations calculated here (e.g., 5.5% for $G_{\rm Ep}$ at ρ_0) and not sufficient to reject them.

In conclusion, we have calculated the density-dependent electromagnetic form factors of a bound "nucleon" within the QMC model. The differential cross section for the eN scattering can be easily constructed once the kinematics is selected. This work represents, to our knowledge, the first quantitative, density-dependent study within a quark model which reproduces the saturation energy, density and compressibility of nuclear matter. While the deviations of the form factors from their free values are within current experimental limits, they provide a strong motivation for forthcoming quasielastic electron-nucleus scattering experiments.

We would like to thank I. Sick for helpful correspondence concerning the constraints from y-scaling. This work was supported by the Australian Research Council. A.W.T and K.S. acknowledge support from the Japan Society for the Promotion of Science.

REFERENCES

[1] J. D. Walecka, Ann. Phys. (N.Y.) **83**, 497 (1974); B. D. Serot and J. D. Walecka, Adv. Nucl. Phys. **16**, 1 (1986).

[2] R. Brockmann and R. Machleidt, nucl-th/9612004, to appear in "Open Problems in Nuclear Matter", ed. M. Baldo *et al.* (World Scientific, Singapore, 1997).

[3] P. A. M. Guichon, Phys. Lett. B **200**, 235 (1988).

[4] S. Fleck, W. Bentz, K. Shimizu, and K. Yazaki, Nucl. Phys. **A 510**, 731 (1990).

[5] K. Saito and A. W. Thomas, Phys. Lett. B **327**, 9 (1994); Phys. Rev. C **52**, 2789 (1995); *ibid.*, C **51**, 2757 (1995); K. Saito *et al.*, Phys. Rev. C **55**, 2637 (1997).

[6] I. Sick, in Proc. Int. Conf. on Weak and Electromagnetic Interactions in Nuclei, ed. H. Klapdor (Springer-Verlag, Berlin, 1986) p.415.

[7] D. B. Day, J. S. McCarthy, T. W. Donnelly and I. Sick, Ann. Rev. Nucl. Part. Sci., **40**, 357 (1990).

[8] J. Jourdan, Phys. Lett. **B353**, 189 (1995).

[9] P. A. M. Guichon, K. Saito, E. Rodionov and A. W. Thomas, Nucl. Phys. **A 601**, 349 (1996); K. Saito, K. Tsushima, and A. W. Thomas, *ibid.* **A 609**, 339 (1996).

[10] P. G. Blunden and G. A. Miller, Phys. Rev. C **54**, 359 (1996);

[11] X. Jin and B. K. Jennings, Phys. Lett. B 374, 13(1996); X. Jin and B. K. Jennings, Phys. Rev. C 54, 1427 (1996); *ibid.*, 55, 1567 (1997).

[12] R. Aguirre and M. Schvellinger, Phys. Lett. B **400**, 245 (1997); P. K. Panda, A. Mishra, J. M. Eisenberg, and W. Greiner, nucl-th/9705045.

[13] A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn and V. F. Weisskopf, Phys. Rev. D**9**, 3471 (1974); A. Chodos, R. L. Jaffe, K. Johnson and C. B. Thorn, Phys. Rev. D**10**,

2599 (1974); T. A. DeGrand, R. L. Jaffe, K. Johnson and J. Kiskis, Phys. Rev. D**12**, 2060 (1975).

[14] E. Naar and M. C. Birse, Phys. Lett. **B305**, 190 (1993); M. K. Banerjee, Phys. Rev C **45**, 1359 (1992).

[15] T. De Forest, Jr., Nucl. Phys. **A392**, 232 (1983); G. I. Poulis, nucl-th/9703054.

[16] D. H. Lu, A. W. Thomas, and A. G. Williams, nucl-th/9706019; D. H. Lu, A. W. Thomas, and A. G. Williams, Phys. Rev. C **55**, 3108 (1997).

[17] R. E. Peierls and D. J. Thouless, Nucl. Phys. **38**, 154 (1962).

[18] A. L. Licht and A. Pagnamenta, Phys. Rev. D**2**, 1156 (1970); X. Ji, Phys. Lett. B **254** (1991).

[19] A. W. Thomas, Adv. in Nucl. Phys., **13**, 1 (1984).

[20] S. Théberge, G. A. Miller, and A. W. Thomas, Can. J. Phys. **60**, 59 (1982); S. Théberge and A. W. Thomas, Nucl. Phys. **A393**, 252 (1983).

[21] G. E. Brown and M. Rho, Phys. Rev. Lett. **66**, 2720 (1991).

FIGURES

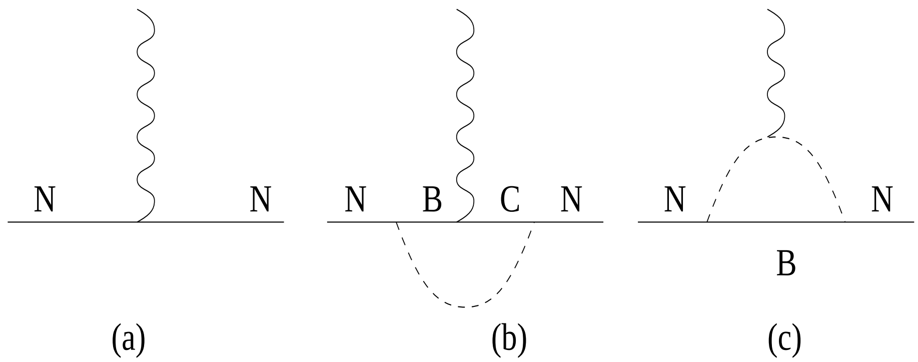

FIG. 1. Diagrams illustrating the various contributions included in this calculation (up to one pion loop). The intermediate baryons B and C are restricted to the N and Δ.

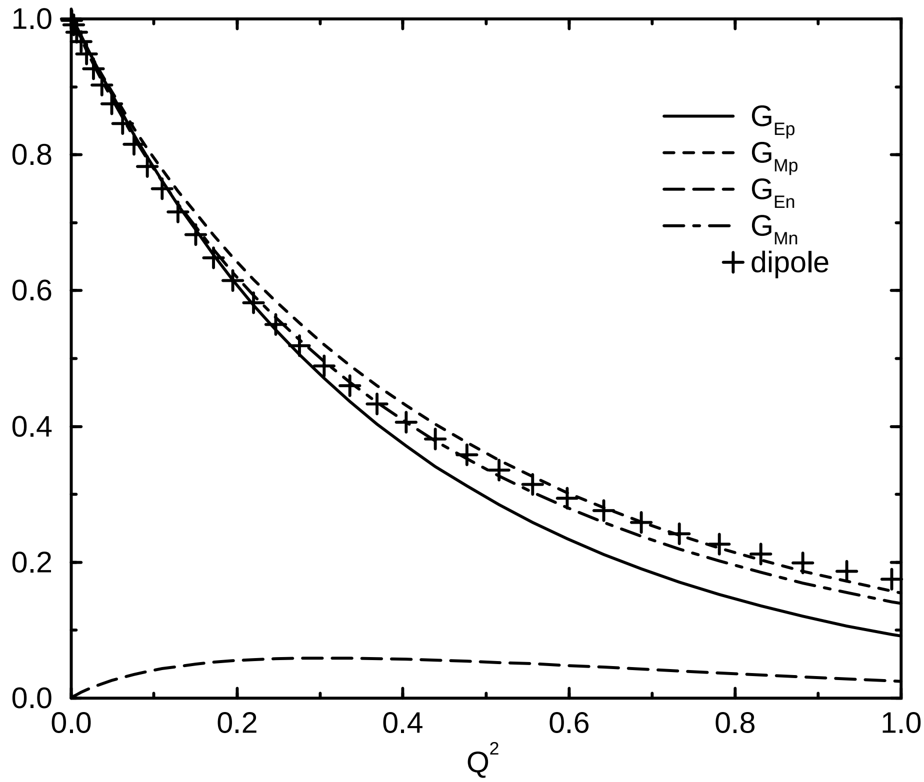

FIG. 2. The nucleon electromagnetic form factors in free space. The bag radius was chosen to be R = 1 fm here.

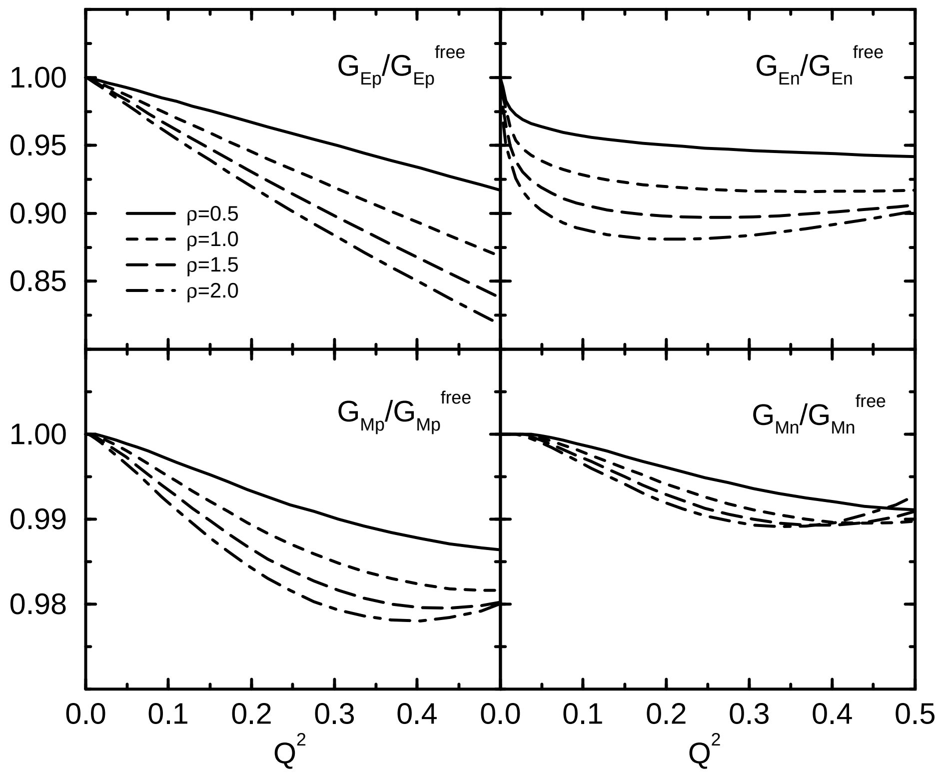

FIG. 3. The nucleon electromagnetic form factors in the nuclear medium (relative to those in free space case). The free space bag radius is 1 fm and the density is quoted in units of the saturation density of symmetric nuclear matter ($\rho_0 = 0.15$ fm^{-3}).